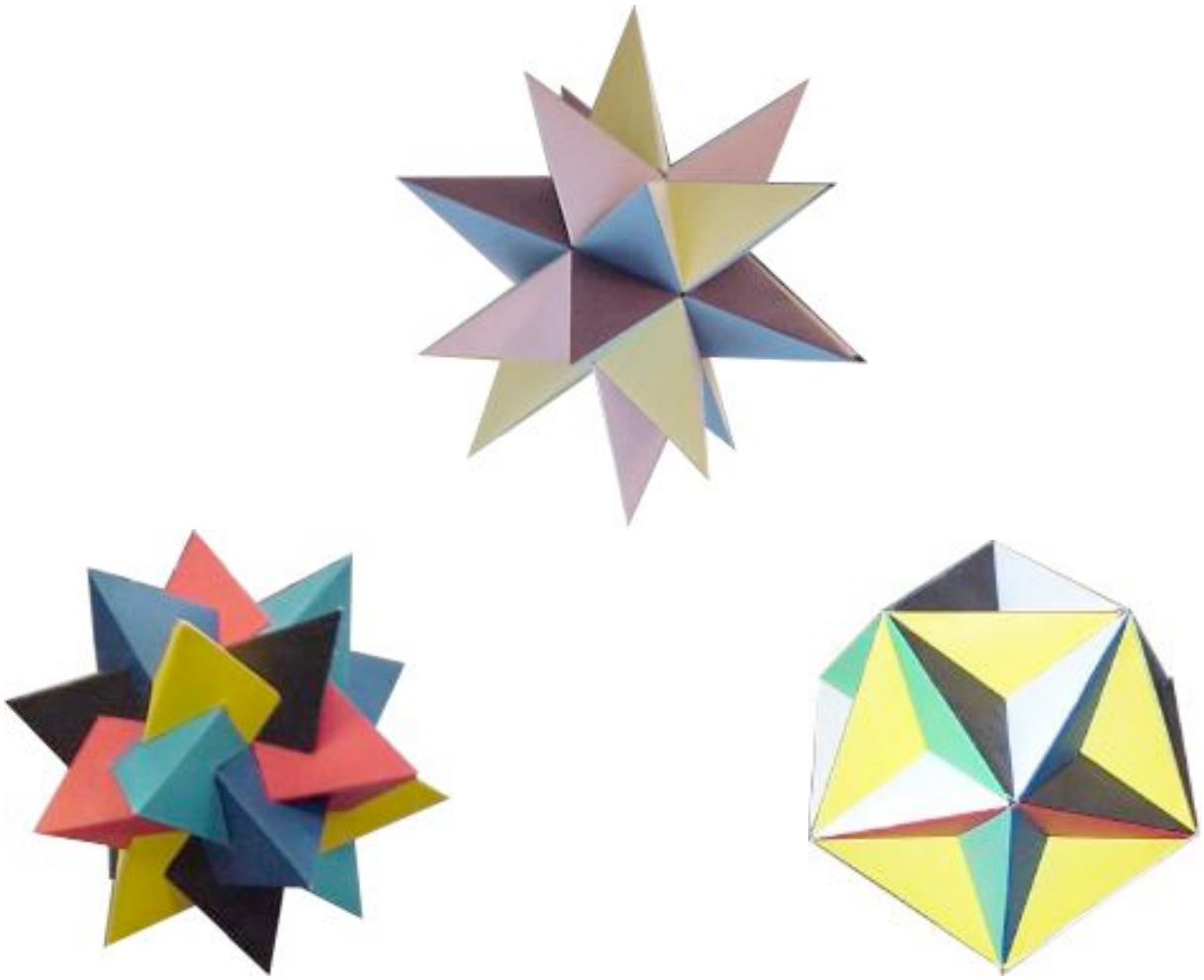
Investigations with Polyhedra



When to use this project: In schools the subject of geometry, whether plane or solid, is often treated in a very abstract way. During this project students are motivated by the desire to create one of these beautiful figures and in doing so, they have hands-on experience with nets and solid geometry concepts.

The applications of this construction activity are many. Our earth science classes have coordinated units on crystal structure that meld well with this activity. Students learn to make polyhedral examples of various crystal shapes. Real-world networking uses notions of 3-dimmensional connectivity. Molecular structure is another topic that can be illustrated well with polyhedra.

Appropriate for students in 7th through 12th grades.

Vocabulary and concepts

polygon
polyhedron, polyhedra
regular polygon
regular polyhedra
truncation

stellation
compound
enanomorphic
face
edge
vertex

Motivation

I generally begin this project after the winter vacation. Students have more indoor time in the winter. The anticipation for this project is another motivation for my students. In our classroom and around the school, there are lovely, student-made polyhedra dangling by fishing line or double strands of sewing thread from the braces between our ceiling tiles. These hanging polyhedra represents years of gifts from students to me or favorite teachers or favorite places in our school. The possibilities for the students are clear.

Background necessary for students

Several weeks before the projects begin, I try to make one polyhedra during team time or in the morning before classes begin so that students can observe my progress, the technique, and ask me questions about my creation.

We do the actual construction in an opening 2 hour-long, block-scheduled class. But, prior to that class, students need to understand the vocabulary and appreciate the possibilities. About a week before the construction day, I show them the five Platonic Solids: tetrahedron, hexahedron, octahedron, dodecahedron and icosahedron. I indicate faces, regular faces (equal angles and equal sides), edges, vertices, and consistent interior angles. It's a good idea to have many models in student hands around the classroom.

I like to get kids to wonder about why there are only 5 possible regular constructions.

Lesson 1: About one week before construction

To allow students to understand why there are only 5 regular solids, I have created sets of cardboard equilateral triangles, squares, pentagon and hexagons. Pattern blocks can also be used. Students equipped with these regular figures can investigate these questions.

1. How many different ways can you place equilateral triangles together to create the vertex of a polyhedron?

Students find that 3 triangles held together make a very sharp point; 4 triangles make a less sharp vertex and 5 triangles held together make a rather dull vertex angle.

However, 6 triangles don't create a vertex angle at all. 6 triangles together lay flat.

Appropriate observations and review: What is the angle measure of one of your equilateral triangles? When you place 6 together what is the total angle measure of their union? Why do you suppose 6 triangles together don't make a vertex

angle of a polyhedra at all?

2. What would the figures look like if the vertex angle that you have just created were consistent throughout the figure? Students discover tetrahedron, octahedron, and icosahedron.

3. Now ask students to build a vertex with squares. They can only build a corner with 3 squares. What would a polyhedron made with vertices containing 3 squares each look like? Students discover a hexahedron = cube.

Can you put 4 squares together in a vertex? What would the sum of four right angles be if you could create a vertex angle with them?

4. Now build vertices with pentagons. One interior vertex angle of a pentagon is 108 degrees. Review how you can figure out this measurement. Three pentagons together can be constructed to form a polyhedral vertex but four pentagons together have a vertex angle that adds up to more than 360 degrees making a concave vertex. What completed figure could you build with pentagons where every vertex had 3 adjacent pentagons?

Students realize dodecahedron.

5. Now try to build vertices with hexagons. Each interior angle of a hexagon has 120 degrees. Three hexagons together create a flat vertex - no polygon.

Therefore, there are only 5 regular solids.

Hopefully throughout this investigation, words like vertices, faces, edges, regular angles will begin to be commonly understood.

Lesson 2: Familiarity with terms and notions of augmentation

Before the actual construction class, students need to become familiar with the new vocabulary and concepts. I like to distribute my Platonic models and ask students to identify the following;

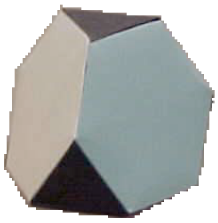
1. The name of the model
2. The shape of the faces in the model.
3. The number of faces in the model
4. The number of vertices in the model
5. The number of edges in the model

*** Euler's formula makes an interesting insert here ***
vertices + faces - edges = 2

This counting encourages students to explain to their classmates how they considered the model as they counted the number of edges, for instance, on an icosahedron. They also have the opportunity to use the terms and become familiar with their meanings.

By now students are familiar with the five Platonic solids and understand the meaning of regular figures. It is time to introduce semi-regular figures. Semi-regular polyhedra are those that have regular faces but also contain more than one kind of regular polygon. There are several types of semi-regular figures … 13 in all.

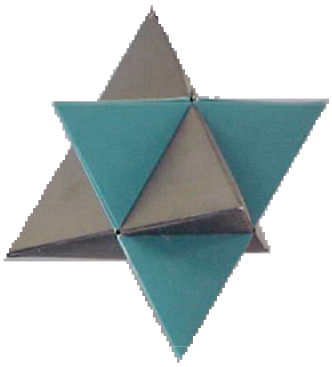
Truncation: Getting students to understand truncation is not difficult if you have a model of a tetrahedron and a model of a truncated tetrahedron or a cube and a truncated cube.



Truncating Platonic solids produce 5 new figures. Each of the 5 Platonic solids can be truncated. There are 8 other semi-regular convex polygons that are created by slicing sides and vertices again. But there are only 13 possible semi-regular polyhedrons. This is a very deep investigation but an option for a motivated student.

Combining two or more polyhedra that appear to intersect each other can create more polyhedra. These are called **Compounded** polyhedra**.** Again, show models.

intersection of an octahedron and a cube

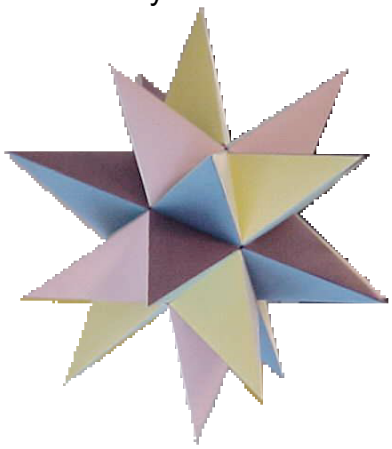


two intersecting tetrahedra

five intersecting tetrahedra

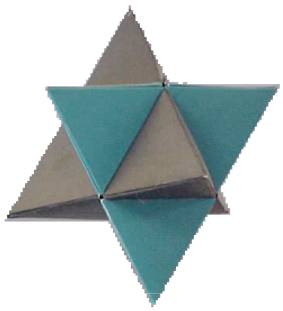
The next important concept is the augmentation of the Platonic solids by **stellation**. If we extend the edges of all of the faces on an icosahedron, we obtain protrusions on each face. These protrusions would look like little pyramids attached to each face = **stellations**.

Show stellated models. Point to the steepness of the stellation decided by the extended edges of the core polyhedra. A lovely stellation is the stellated icosahedron.

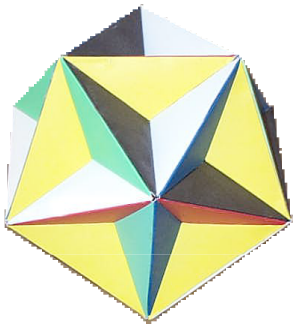


At this point I like to show how the coloring of the models can emphasize different aspects of it. On the stellated icosahedron, I can use only one color to create a star or I can use six colors to emphasize the **parallel facial planes** and the two-d pentagrams that can be seen from each angle.

A stellated model of an octahedron can be colored to look like an eight pointed star or done in two colors, as below, to look like two intersecting tetrahedron.



Lastly, two other polyhedra producing techniques should be shown. If a dimple (indentation) were placed on each face of a regular polyhedron, I might obtain a very different looking figure. The Great Dodecahedron can be viewed as a dimpled icosahedron.



Lesson 3 - Day of construction

Supplies:

Ask students to gather at home and bring to class on this day:

- Thin cardboard (not corrugated), from back of pads of paper or from cereal boxes, shirt cardboard, etc.
- ball point pens
- straight edges
- old magazine to be used under the template and construction paper in the vertex stabbing procedure.

I supply;

- compass points or T-pins
- school scissors
- white glue
- construction paper
- bobby pins (we call them “math clamps”)

I ask students to bring their own scissors if they prefer to work with tools that are more precise than my school scissors.

Objective: Students will create a Platonic Solid in order to learn the technique of polyhedra construction. Their actual project will be due 10 days to two weeks later and is somewhat restricted in scope.

1. I have students gather around a set of tables that have been pushed together to observe the technique. I show them a perfect triangle that I have created with straight edge, compass, and scissors or on the computer with a drawing program that I will use as my

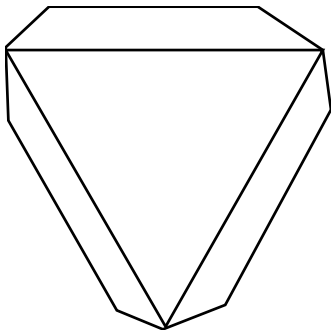
template. I glue the paper triangle onto the thin cardboard and suggest that they consider three ways to think about cutting out their template; leave the line on, cut the line off, or split the line. I suggest that they try to cut out their template by barely leaving their line on.

Their templates will not be traced around. To make a good model, the template cannot be damaged during the construction. So, I show them how to mark the vertices with a hole poked just outside of their template with compass point on their construction paper (backed with a magazine) and then to connect the vertices dots with straight edge and ballpoint pen. The purpose of using the ballpoint pen is to score the construction paper and allow for perfect folds.

Students who will be making models involving other faces than a triangle will need to have faces produced from the computer to insure regular construction and then glued onto cardboard backings. The final cropping of their template (leaving the line on but just barely) should not be done until the glue has dried.

When the template is dried and trimmed (leaving the line on), I demonstrate the vertex punching, pen and straight-edge scoring, cutting out construction paper piece with tabs left on each side and the folding.

Once a construction paper triangle has been drawn. I show students that they need to cut out the face but leave on enough material to glue the faces together = tabs or seam allowance.



They also need to understand that the need to bevel the ends of their tabs so that paper won't get crumpled into the corner joints.

3. I glue two faces together and point out the necessity of keeping vertices aligned and looking at the good side of the joint for evenness in the facial planes. I clamp the glued surfaces together with "math clamps" (bobby pins) and set the piece aside for 5 minutes. I alternate gluing and cutting in order to allow joints to dry.

4. When affixing the final face of the polyhedron, I ask students to think of closing the lid of a box. We glue one hinge joint, let it dry, and then glue both of the remaining joints at the same time as we lower the lid to the box and hold it for 5 minutes - Voila !

Students then choose one of the five Platonic solids to try to create. The technique requires patience and good fine motor ability. I try to encourage students to expect a poor first construction. This first construction is merely to learn the technique and become accustomed to the timing and process required. Construction begins. Students generally help each other and also require my help. This is a very social and pleasurable one-hour experience.

Assignment: Students are assigned to build (in accordance with their enthusiasm and fine motor talent) any model that they see in the room or on the posters (other than the Platonic Solids). I often help them create templates with the computer's ability to make regular polygons of any size and number of sides. In building a polyhedra that contains, for example, octagons and triangles (a truncated hexahedron) it is necessary to create two templates ... one, of an octahedron with the same size side as a second template, a triangle.

Assessment: Students are very concerned about the pressure of making a detailed, elaborate, time-consuming construction. Fine motor ability and art project patience varies dramatically from student to student. During the 2-hour construction day, it is easy to note who is talented at accurately and neatly gluing joints and which students find this task very difficult.

When students are choosing their projects I make sure that they understand the rubric.

- Your project must be more complex than a platonic solid
- Your project must be neatly assembled
- Your project must be appropriate to your skill level which I have just observed.
- You must use color to show facial planes and parallel planes

(There are often wonderful exceptions to this color requirement. Soccer balls; red stars of good luck; glittery decorations ...)

Polyhedra Assignment

You are assigned to build (according to your fine motor ability and enthusiasm) any model that you see in the room, on the posters, or in my polyhedra book.

You must see me individually to get your polyhedra templates and permission to attempt the figure that you have chosen to complete.

Your project will be graded according to accuracy, neatness, effort expended and color. The difficulty of your project will not affect your grade.

You should begin immediately in order to have enough time to peacefully build your polyhedra allowing for plenty of glue-drying time and time for getting help. Beginning soon will also help you remember the steps that we have practiced in our block schedule class.

Enjoy !