**Golden Ratio Project**

**When to use this project:** Golden Ratio artwork can be used with the study of Ratios, Patterns, Fibonacci, or Second degree equation solutions and with pattern practice, notions of approaching a limit, marketing, review of long division, review of rational and irrational numbers, introduction to \( \varphi \).

**Appropriate for** students in 6th through 12th grades.

**Vocabulary and concepts**
- Fibonacci pattern
- Leonardo Da Pisa = Fibonacci = son of Bonacci
- Golden ratio
- Golden spiral
- Golden triangle
- Phi, \( \varphi \)

**Motivation**
Through the investigation of the Fibonacci sequence students will delve into ratio, the notion of irrational numbers, long division review, rational numbers, pleasing proportions, solutions to second degree equations, the fascinating mathematics of \( \varphi \), and more.

**Introductory concepts**
In the 13th century, an Italian mathematician named Leonardo Da Pisa (also known as Fibonacci -- son of Bonacci) described an interesting pattern of numbers. The sequence was this; 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...
Notice that given the first two numbers, the remaining sequence is the sum of the two previous elements. This pattern has been found to be in growth structures, plant branchings, musical chords, and many other surprising realms. As the Fibonacci sequence progresses, the ratio of one number to its proceeding number is about 1.6. Actually, the further along the sequence that one continues, this ratio approaches 1.618033988749895 and more. This is a very interesting number called by the Greek letter phi \( \varphi \). Early Greek artists and philosophers judged that a
desirable proportion in Greek buildings should be width = \(\phi\) times height. The Parthenon is one example of buildings that exhibit this proportion.

\[
\text{The Greek language symbol for phi is } \phi.
\]

The Greeks thought that this was a pleasing dimension for a building or any structure. It was not too stocky and not too thin. They called this proportion the Golden proportion. Actually they wanted the ratio of the length to the height to be the same as the ratio of the (length plus height) to the length.

That is \(\frac{y}{x} = \frac{x + y}{y}\)

This pleasing proportion is still used. Product marketing often exhibits the \(\phi\) ratio.

First night’s homework:
Find examples in products that you might have at home that approximate this proportion. Bring in either the object or its dimensions. Do you think that this ancient Greek observation is still being used today?

suggestions: cereal boxes, library cards,

(8th, 9th, or 10th grade algebra classes)

In an algebra class you could do the following. Let’s let the height of a golden rectangle be 1 unit. Then our picture would look like this.

\[
\begin{array}{c|c}
1 & x \\
\hline
\end{array}
\]

For this rectangle to exhibit the Golden Ratio, this proportion must be true, \(\frac{x}{1} = \frac{x + 1}{x}\).

To solve this solution for \(x\) we might solve the proportion by cross-multiplying.

\(x^2 = x + 1\) or \(x^2 - x - 1 = 0\)

Solving this equation with the quadratic formula, students would find that \(x\) must equal \(\frac{1 \pm \sqrt{5}}{2}\).

Now \(\frac{1 - \sqrt{5}}{2}\) is a negative number because \(\sqrt{5}\) is larger than 1. So \(\frac{1 - \sqrt{5}}{2}\) is meaningless as the side of a rectangle. Therefore, the only possible solution must be \(\frac{1 + \sqrt{5}}{2}\). This is the Golden
ratio which is called \( \varphi \). Students can evaluate this ratio with their calculators and get about 1.618033989.

Now in the Fibonacci sequence, I could begin testing succeeding ratios
\[
\frac{1}{1} = 1 \\
\frac{2}{1} = 2 \\
\frac{3}{2} = 1.5 \\
\frac{5}{3} = 1.6 \\
\frac{8}{5} = 1.6 \quad \text{This is a great long division recap.} \\
\frac{13}{8} = 1.625 \\
\frac{21}{13} = 1.615384 \\
\frac{34}{21} = 1.619047 \\
\frac{55}{34} = 1.61764705882352941
\]

There is a clear approach to a number about 1.6. It is easy to show that the first division is less than \( \varphi \), the next division is more than \( \varphi \), the next division is less than \( \varphi \), and so forth. Other teachable moments continue to appear in this discussion.

• “But my calculator says that \( \frac{21}{13} = 1.6153846154 \). What made you think that it is a repeating decimal?”

• “How did you know to keep dividing 55 by 34 until you found a repeat?”

• “I don’t remember how to divide.”

Now lets start to look at what the Fibonacci sequence can look like.

![Fibonacci Spiral]

The Fibonacci sequence can be viewed as a spiral by drawing successive Fibonacci-sized squares. Start with a 1 x 1 square in the center of your paper. Beside that square place another 1 x 1 square. Since 1 + 1 = 2, the next square will be a 2 x 2. 1 + 2 is 3. So, the next square is 3 x 3, and so on. Each succeeding square is placed in a counter-clockwise position to the last square as they rotate around the initial 1x1 square. Show students that rectangles are formed when you combine a new square with all of the previous squares.
These rectangles have a length to width ratio that approaches the Golden Ratio just as we saw in the list of Fibonacci ratios on the previous page.

If within each square, a quarter circle is drawn with the circle’s center being the corner of the square closest to the center of the pattern, a spiral is created. This is called the Golden Spiral.

The Fibonacci pattern can also be used to create interesting images that seem to be distortions of space. Here’s another representation of the Fibonacci pattern. In our school this is the background of the girl’s soccer shirts.

Students have created lovely artwork by coloring this lattice or emphasizing Golden Rectangles that can be found within it. In this lattice, the center of the rectangle is where the pattern begins. The rectangle's width is divided into lengths from the center out according to the Fibonacci pattern.

The following quilt was inspired by a student's work. The bold colors of turquoise and orange really help the viewer to notice the distortion that this pattern brings to view.
Golden Triangles

As you would expect, the base of a Golden Triangle multiplied by $\phi$ will equal the sides of this isosceles triangle. This exact triangle can be found inscribed in a regular pentagon.

Compass and straight edge will allow students to create wonderful iterative designs with these two images. The diagonal of a pentagon is $\phi$ times the length of one side.

Lute of Pythagoras

The basis of this design is the Golden Triangle. A triangle is created with the ratio of isosceles sides to the base of phi. In other words, the length of the triangle sides is about 1.618 times larger than the length of the base. One can create an enclosed Golden Triangle by duplicating the base length and rotating it clockwise 36 degrees. Using a compass helps students do this base size duplication easily. Other Golden Triangles can be formed by rotating the base length in a counter clockwise rotation and by drawing lines that are parallel to the base.
By continuing to connect vertices, one begins to find pentagrams (five pointed stars) and pentagons throughout the figure.

The Golden Ratio, phi, can be found repeatedly in pentagons and pentagrams. Any diagonal of a pentagon is phi times larger than the side of the pentagon. The length of one star point of a pentagram is phi times the interior pentagon’s side or the base of the Golden Triangle that is the star’s point.

Other fabulous \( \phi \) facts;

- \( \phi \) is about 1.618033988749895
- \( 1/\phi \) is about .618033988749895

Do you notice anything cool with the two values above?

\[
\phi - 1 = \frac{1}{\phi}
\]

Since \( \phi = 1 + \frac{1}{\phi} \) then \( \phi = 1 + \frac{1}{1 + \frac{1}{\phi}} = 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{\phi}}} \) etcetera
Assignment sheet:

Golden Ratio Art Project

Using the Golden Ratio or the Fibonacci pattern demonstrate using art, music, nature or architecture, the interesting possibilities of the ratio phi.

Your project will be graded according to these guidelines;

1. Your project will clearly express the Golden Ratio. If it is not visually clear than you will explain your observation of the ratio in written form.

2. Your project will be attractive.

3. Your project will be neat.

4. Your project will have color and/or texture.

5. It will be clear to me that you have given thought and energy to this assignment.

6. No part of your project will be cut and pasted from the internet.